

Merging MEs & PS in **SHERPA**

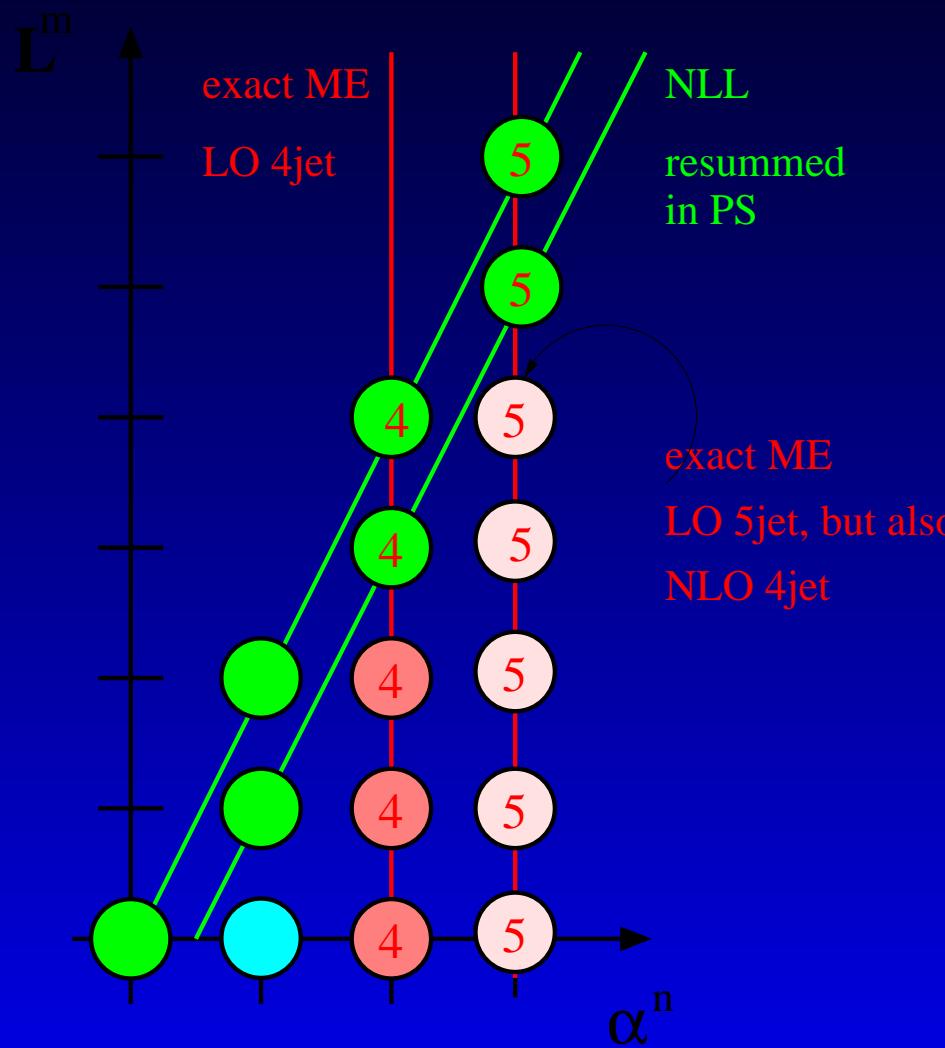
Simulation for High Energy Reactions of PArticles

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ME vs. PS

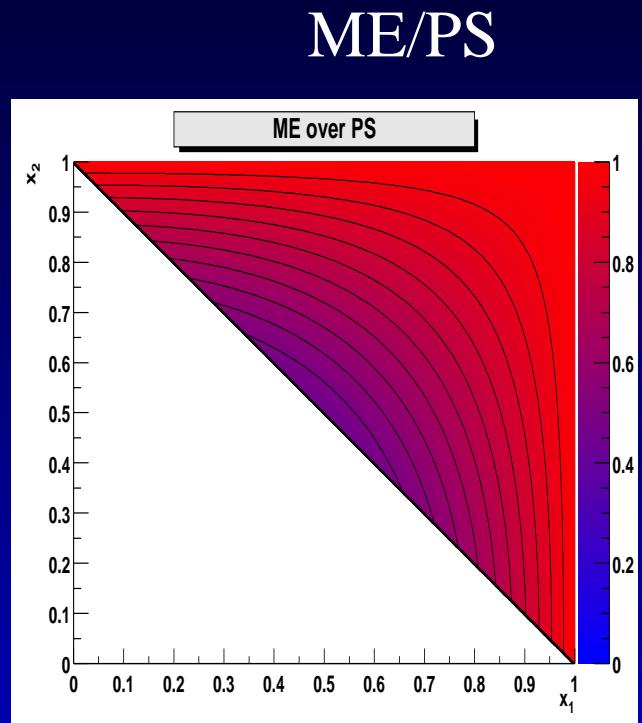
- ME is **exact** at some given order in the coupling constant α_s etc..
 \implies ME is “exclusive”.
- PS **resums** all orders of $\alpha_s^n (L^{2n} + L^{2n-1})$.
 L are large logs like $\ln s/Q_0^2$
 \sqrt{s} is cm energy, Q_0^2 is some infrared scale.
 \implies PS is “inclusive”.



Example : $e^+e^- \rightarrow q\bar{q}g$:

$$\frac{d\sigma_{ME}}{dx_1 dx_2} \propto \left| \begin{array}{c} \text{Feynman diagram 1} \\ + \\ \text{Feynman diagram 2} \end{array} \right|^2$$

$$\frac{d\sigma_{PS}}{dx_1 dx_2} \propto \left| \begin{array}{c} \text{Feynman diagram 1} \\ + \\ \text{Feynman diagram 2} \end{array} \right|^2$$



Brief Review of Parton Shower

“Parton parted partons”

Strategy : Consider soft & collinear regions (Bulk of the emission)

Consider production of light extra particle ($N \rightarrow N + 1$):

$$d\sigma_{N+1} \sim d\sigma_N \int \frac{dt_a}{t_a} \cdot P_{a \rightarrow bc}(z)$$

- Propagator $1/t_a$ for decaying particle a :
 $t_a = p_a^2 \approx 2E_b E_c(1 - \cos \theta_{bc})$.
- Splitting function, decay characteristics For emitted massless bosons (gluons, photons): $P_{a \rightarrow bc}(z) \xrightarrow{z \rightarrow 0} \infty$

Brief Review of Parton Shower

“Parton parted partons”

Resummation of these logs possible

⇒ Sudakov form factor $\Delta(T, t)$:

$$\Delta(T, t) = \exp \left\{ - \int_{t'}^T \frac{dt'}{t'} \int_{z_-(t')}^{z_+(t')} dz \frac{\alpha[p_\perp^2(z, t')]}{2\pi} P_{a \rightarrow bc}(z) \right\}$$

Interpretation (simplified) : $\Delta(t, t_0)$ = “no-branch” probability.

ME + PS for e^+e^-

S. Catani, F. Krauss, R. Kuhn, B. Webber, JHEP 0111:063,2001

Goal: Universality of fragmentation (energy dependent)

Goal: Combination procedure of ME + PS:

Produce jets (energetic particles, large angles) by ME

Evolve jets by PS down to fragmentation scale

Strategy: Divide phasespace into jet production & jet evolution

$$2 \cdot \min\{E_i^2, E_j^2\} \cdot (1 - \cos \theta_{ij}) \stackrel{>}{<} y_{\text{jet}} E_{\text{cm}}^2$$

Weight ME's, Veto on PS.

Step 1: NLL-weighted MEs

Consider jet rates at next-to leading log accuracy $Q_{\text{jet}} = y_{\text{jet}} E_{\text{cm}}$:

$$\mathcal{R}_2(Q_{\text{jet}}, E_{\text{cm}}) = [\Delta_q(Q_{\text{jet}}, E_{\text{cm}})]^2$$

$$\mathcal{R}_3(Q_{\text{jet}}, E_{\text{cm}}) = 2 [\Delta_q(Q_{\text{jet}}, E_{\text{cm}})]^2 \int_{Q_{\text{jet}}}^Q dq \Gamma_q(q, E_{\text{cm}}) \Delta_g(Q_{\text{jet}}, q)$$

$\Delta(p, q)$ = No branch between p and q (Sudakov form factor),

$\Gamma(q)$ = one branch at q including coupling constant etc.

Step 1: NLL-weighted MEs

$$\mathcal{P}_{q \rightarrow qg}(z) = C_F \frac{1+z^2}{1-z} = C_F \left[\frac{2}{1-z} - (1+z) \right]$$

\implies Sudakov form factor in NLL

$$\Delta_q = \exp \left[- \int_{Q_{\min}}^{Q_{\max}} dq \Gamma_{q,g}(q, Q_{\max}) \right]$$

with new kernel

$$\Gamma_q(q, Q) = \frac{2C_F}{\pi} \frac{\alpha_s(q)}{q} \left(\log \frac{Q}{q} - \frac{3}{4} \right)$$

Step 2: Kinematics

1. Choice of vectors p_i according to M.E. with $\alpha_s(Q_{\text{jet}})$.
2. Look for “shower” kinematics :
 - Merge $p_i + p_j = p_{ij}$ with smallest y_{ij} .
 - Repeat “parton–merging” until core $2 \rightarrow 2$ process.
3. Generate weight with factors
 - $\Delta_{q,g}(Q_{\text{jet}}, Q_i)$ for outgoing partons from Q_i
 - $\Delta_{q,g}(Q_{\text{jet}}, Q_j)/\Delta_{q,g}(Q_{\text{jet}}, Q_i)$ for propagators $Q_j \rightarrow Q_i$
4. Reweight for different scales in α_s by factors
 $\alpha_s(Q_i)/\alpha_s(Q_{\text{jet}})$.

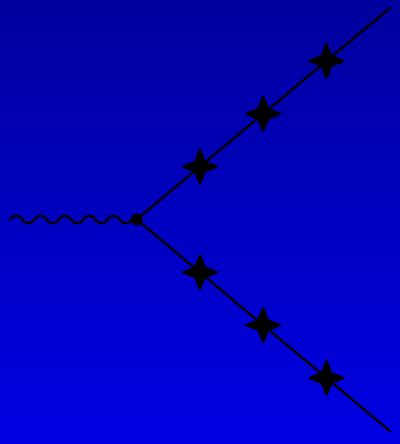
Step 3: Vetoed showers

- Naive guess : Start shower at Q_{jet} .
- Consider $\mathcal{R}_2(Q_0 < Q_{\text{jet}})$.

Starting from Q_{jet} : $[\Delta_q(Q_0, Q_{\text{jet}})\Delta_q(Q_{\text{jet}}, Q)]^2$

instead of : $[\Delta_q(Q_0, Q)]^2$

- Better : Start shower at Q and veto all $q > Q_{\text{jet}}$.

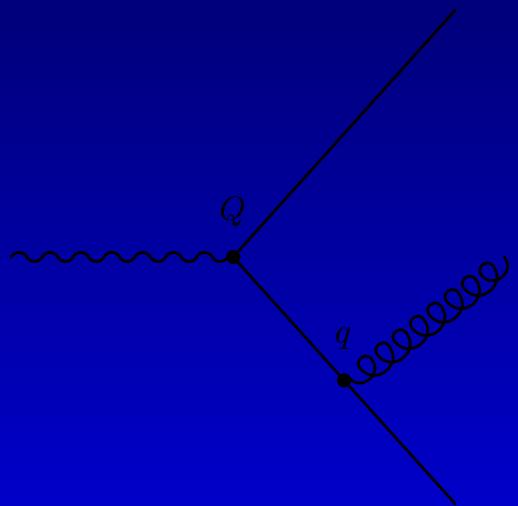


Sum for one quark-line :

$$\Delta_q(Q_0, Q)\Delta_q(Q_{\text{jet}}, Q) \left[1 + \int_{Q_{\text{jet}}}^Q dq \Gamma_q(q, Q) + \dots \right]$$
$$\Rightarrow \frac{\Delta_q(Q_0, Q)\Delta_q(Q_{\text{jet}}, Q)}{\Delta_q(Q_{\text{jet}}, Q)} = \Delta_q(Q_0, Q).$$

Correct Jetrates ?

$$\begin{aligned}\mathcal{P} &= 2\Delta_q(Q_{\text{jet}}, Q) \frac{\Delta_q(Q_{\text{jet}}, Q)}{\Delta_q(Q_{\text{jet}}, q)} \\ &\quad \Gamma_q(q, Q) \Delta_q(Q_{\text{jet}}, q) \Delta_g(Q_{\text{jet}}, q)\end{aligned}$$



$\implies \mathcal{R}_3$ after $\int dq$ ($Q_{\text{jet}} \leq q \leq Q$).

Improvement : Replace Γ_q with
full ME squared.

\implies Better description of region
 $y_{qg}, y_{\bar{q}g} > y_{\text{jet}}$.

Correct Jetrates ?

For three jets : Two possibilities.

Either :

2 jet at Q_{jet} + **one branch**



$$2 [\Delta_q(Q_{\text{jet}}, Q)]^2 \left[\frac{\Delta_q(Q_0, Q)}{\Delta_q(Q_{\text{jet}}, Q)} \right]^2$$

$$\int_{Q_0}^{Q_{\text{jet}}} dq \Gamma_q(q, Q) \Delta_g(Q_0, q)$$

Or :

3 jet at Q_{jet} + **no branch**



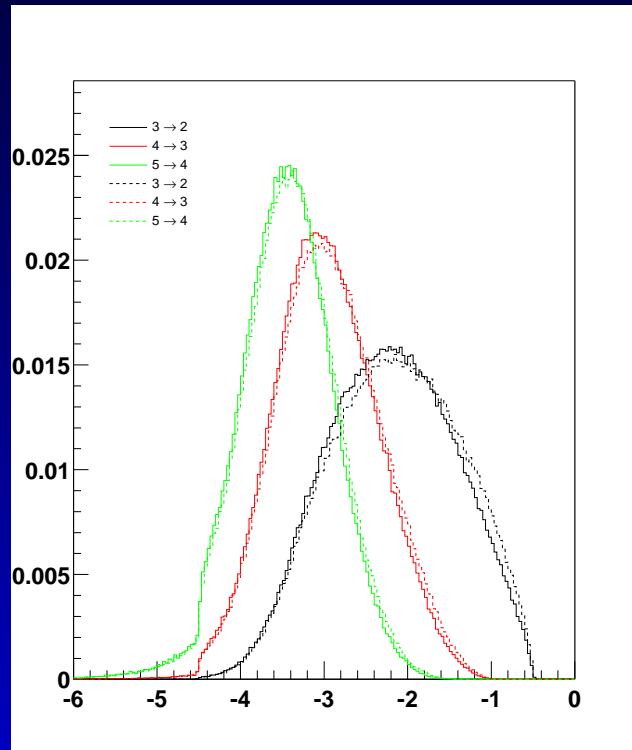
$$2 [\Delta_q(Q_{\text{jet}}, Q)]^2 \left[\frac{\Delta_q(Q_0, Q)}{\Delta_q(Q_{\text{jet}}, Q)} \right]^2$$

$$\int_{Q_{\text{jet}}}^Q dq \Gamma_q(q, Q) \Delta_g(Q_{\text{jet}}, q) \frac{\Delta_g(Q_0, Q)}{\Delta_g(Q_{\text{jet}}, Q)}$$

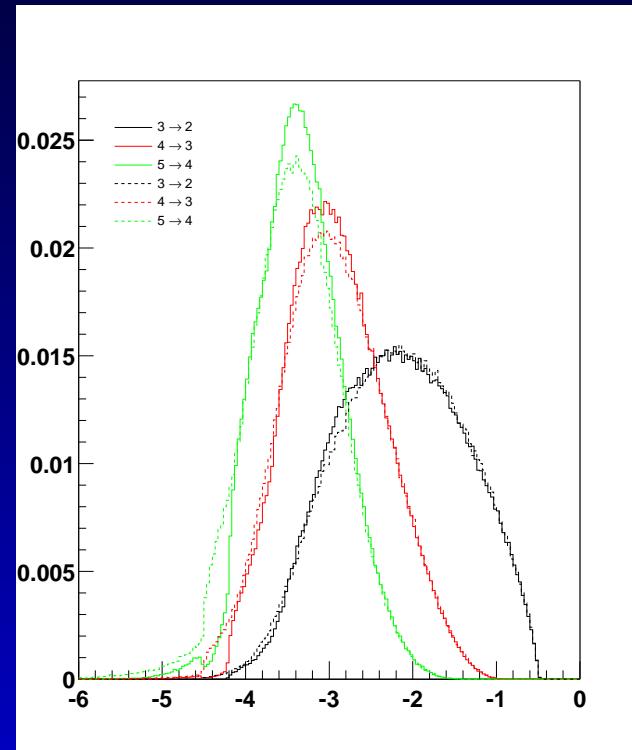
Parton level (APACIC++ 2.0)

Pure parton shower

ME correction



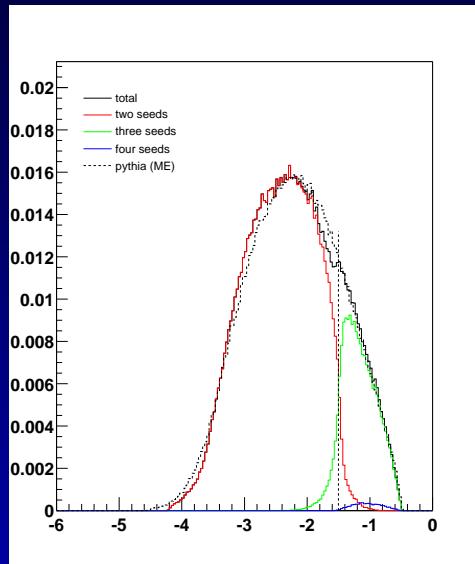
APACIC vs. PYTHIA



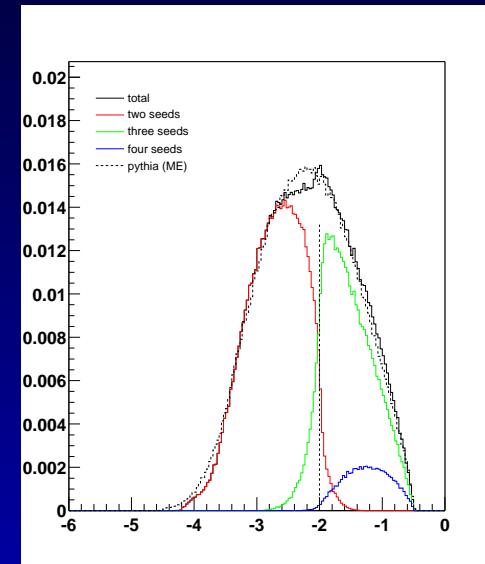
Parton level (APACIC++ 2.0)

Merged parton shower : y_{32}

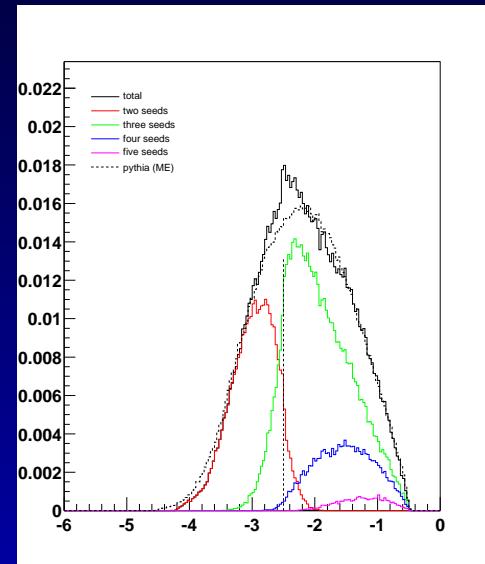
$\log y_{\text{cut}} = 1.5$



$\log y_{\text{cut}} = 2.0$



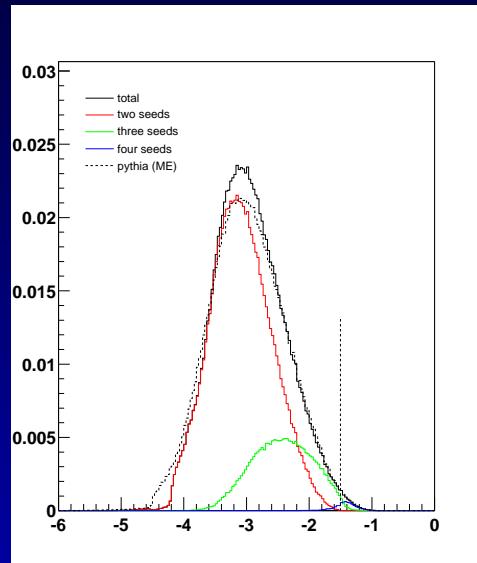
$\log y_{\text{cut}} = 2.5$



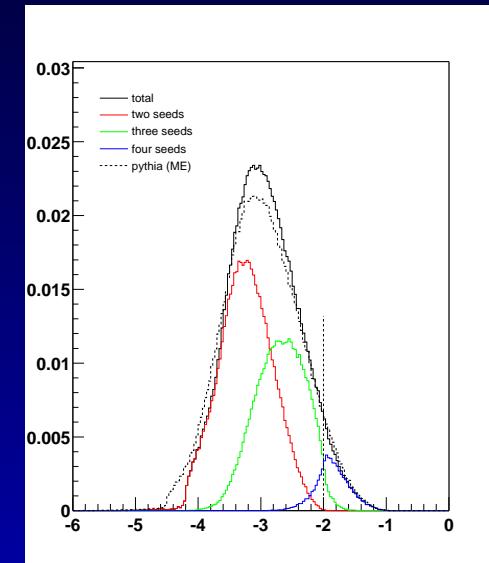
Parton level (APACIC++ 2.0)

Merged parton shower : y_{43}

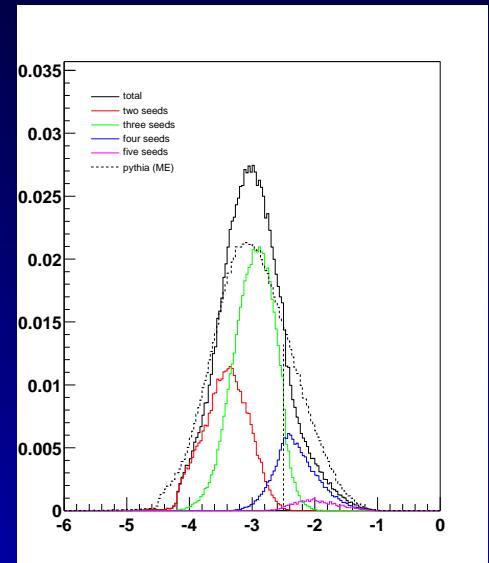
$\log y_{\text{cut}} = 1.5$



$\log y_{\text{cut}} = 2.0$



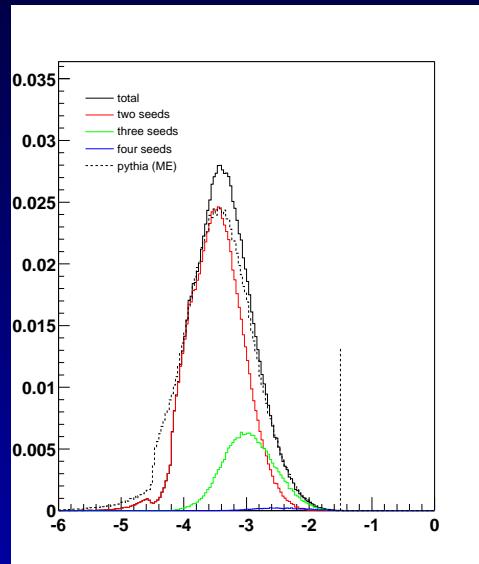
$\log y_{\text{cut}} = 2.5$



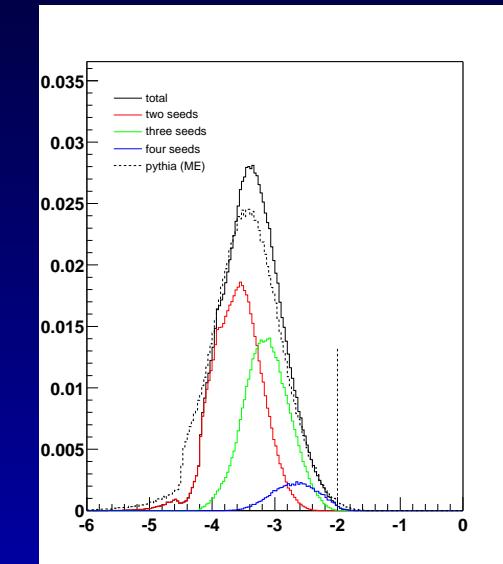
Parton level (APACIC++ 2.0)

Merged parton shower : y_{54}

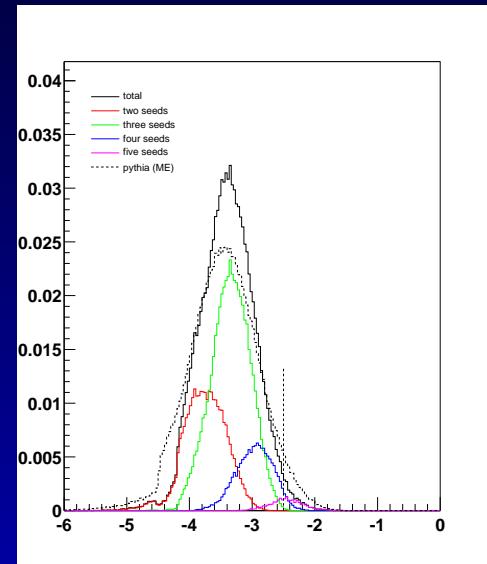
$\log y_{\text{cut}} = 1.5$



$\log y_{\text{cut}} = 2.0$



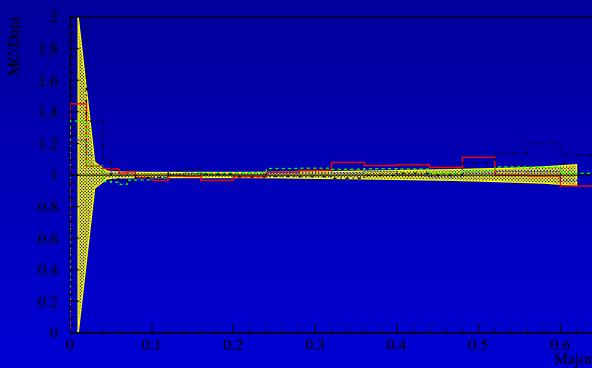
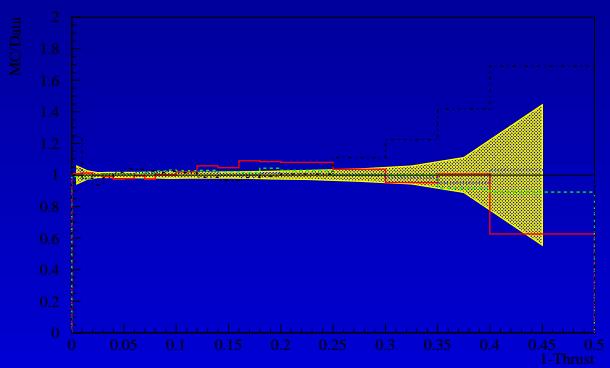
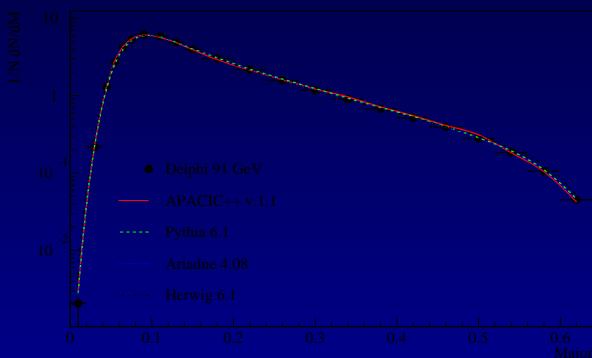
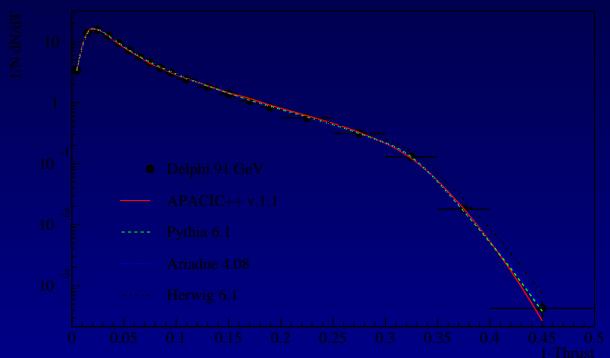
$\log y_{\text{cut}} = 2.5$



Event shapes :

1 - Thrust

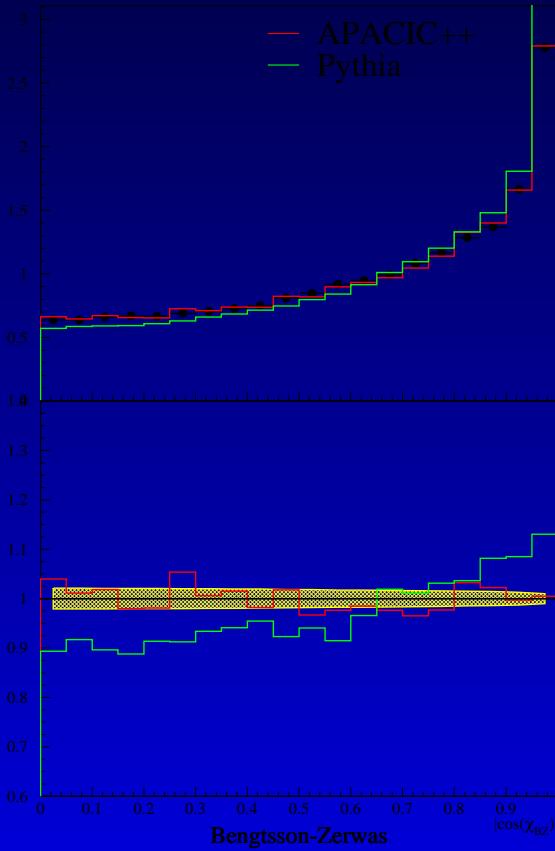
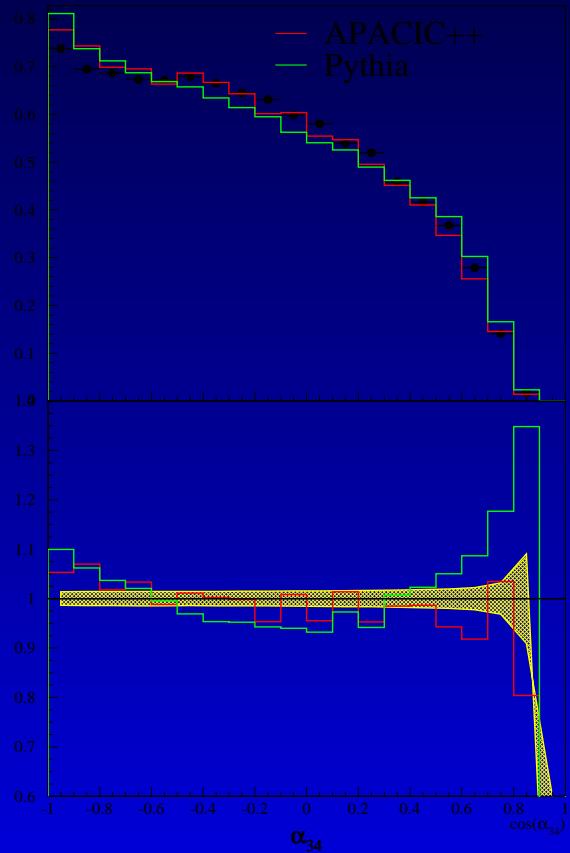
Major



Four jet angles :

α_{34}

Bengtson-Zerwas



Brief review of PS in the IS

- Cross sections:

$$\sigma = \int dx_1 dx_2 f_{1/h_1}(x_1, \mu_F^2) f_{2/h_2}(x_2, \mu_F^2) \hat{\sigma}_{12}(\hat{s}, \alpha_s(\mu^2), Q^2/\mu_F^2)$$

Q^2 is some process-dependent scale (like p_\perp^2)

For simplicity, set all $\mu_F^2 = Q^2$ (to cancel unwanted large logs).

- Backward evolution:

$$\Pi(t_1, t_2; x) = \frac{f(x, t_1)}{f(x, t_2)} \frac{\Delta(t_2, t_0)}{\Delta(t_1, t_0)}$$

yields probability for no branch between t_2 and t_1 resolvable at t_0 .

PDF's in Π ensure correct behaviour at low/high x .

- Select z at scale t of splitting according to $\frac{\alpha(t)}{2\pi} P_{a \rightarrow bc}(z) \frac{x/z f(x/z, t)}{x f(x, t)}$

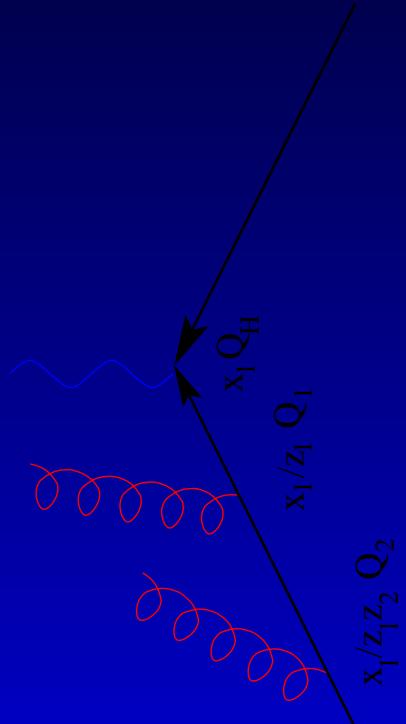
Proposed method :

F. Krauss,JHEP 0208:015,2002

1. Generate n -jets F.S. according to some suitable rate.
2. Cluster F.S. particles backwards according to a k_\perp scheme.
Find the **hardest** k_\perp^2 in the “core” $2 \rightarrow 2$ -subprocess.
Examples: $\hat{s} = M_{ll}^2$ in Drell-Yan type $q\bar{q} \rightarrow l\bar{l}$
 $\frac{2\hat{s}\hat{t}\hat{u}}{\hat{s}^2 + \hat{t}^2 + \hat{u}^2}$ in QCD.
3. Apply a weight constructed by comparing histories.
4. Start initial and final state parton showers from the $2 \rightarrow 2$ -subprocess.
5. Veto on rising k_\perp in subsequent branchings.

Example : Drell-Yan

$$w_{PS} = \frac{f(x_1/z_1, t_1)}{z_1 f(x_1, t)} \frac{\Delta_q(t, t_{jet})}{\Delta_q(t_1, t_{jet})} \cdot P_{q \rightarrow qg}(z_1) \\ \times \frac{f_q(x_1/z_1 z_2, t_2)}{z_2 f_q(x_1/z_1, t_1)} \frac{\Delta_q(t_1, t_{jet})}{\Delta_q(t_2, t_{jet})} \cdot P_{q \rightarrow qg}(z_2) \\ \times \frac{f_q(x_1/z_1 z_2, t_{jet})}{f_q(x_1/z_1 z_2, t_2)} \frac{\Delta_q(t_2, t_{jet})}{\Delta_q(t_{jet}, t_{jet})} \cdot d\hat{\sigma}_{q\bar{q}}(\hat{s})$$



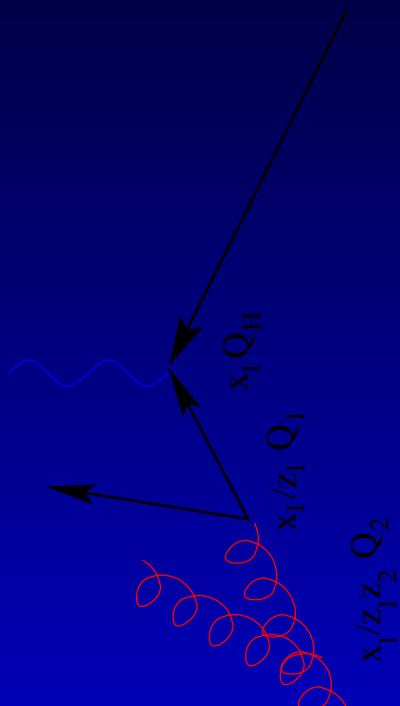
Compare with ($t_{jet} = y_{cut}\hat{s}$):

$$w_{ME} = f_q(x_1/z_1 z_2, t_{jet}) f_{\bar{q}}(x_2, t_{jet}) \\ d\hat{\sigma}_{q\bar{q} \rightarrow ggll}(\hat{s}/z_1 z_2, \alpha_s(t_{jet}), y_{cut})$$

Yields (for upper quark line only, no FS gluons):

$$w_{\text{corr.}} = \Delta_q(t, t_{jet})$$

Example : Drell-Yan

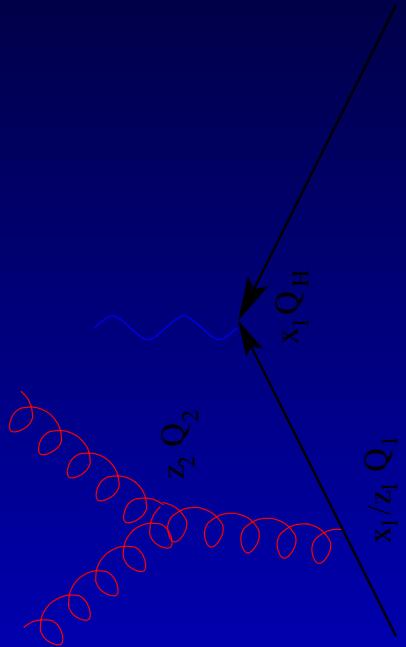


$$\begin{aligned}
 w_{PS} = & \frac{f_g(x_1/z_1, t_1)}{z_1 f_g(x_1, t)} \frac{\Delta_q(t, t_{\text{jet}})}{\Delta_q(t_1, t_{\text{jet}})} \cdot P_{g \rightarrow q\bar{q}}(z_1) \\
 \times & \frac{f_g(x_1/z_1 z_2, t_2)}{z_2 f_g(x_1/z_1, t_1)} \frac{\Delta_g(t_1, t_{\text{jet}})}{\Delta_g(t_2, t_{\text{jet}})} \cdot P_{g \rightarrow gg}(z_2) \\
 \times & \frac{f_g(x_1/z_1 z_2, t_{\text{jet}})}{f_g(x_1/z_1 z_2, t_2)} \frac{\Delta_g(t_2, t_{\text{jet}})}{\Delta_g(t_{\text{jet}}, t_{\text{jet}})} \cdot d\hat{\sigma}_{q\bar{q}}(\hat{s}),
 \end{aligned}$$

yields (for upper line only, no FS particles):

$$w_{\text{corr.}} = \Delta_q(t, t_{\text{jet}}) \cdot \frac{\Delta_g(t_1, t_{\text{jet}})}{\Delta_q(t_1, t_{\text{jet}})}.$$

Example : Drell-Yan

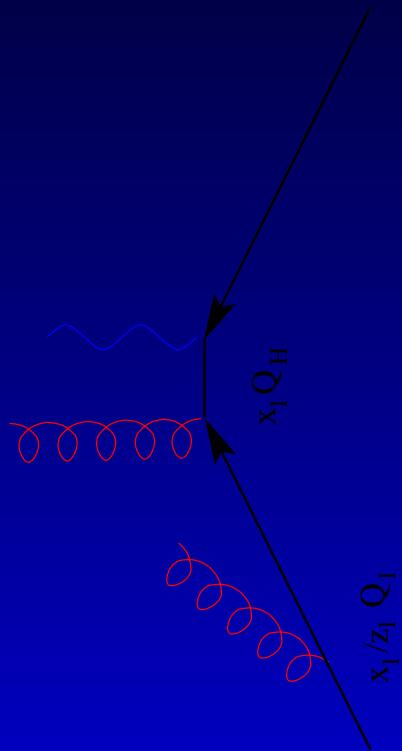


$$\begin{aligned}
 w_{PS} = & \frac{f_q(x_1/z_1, t_1)}{z_1 f_q(x_1, t)} \frac{\Delta_q(t, t_{\text{jet}})}{\Delta_q(t_1, t_{\text{jet}})} \cdot P_{q \rightarrow qg}(z_1) \\
 \times & \frac{f_q(x_1/z_1, t_{\text{jet}})}{f_q(x_1/z_1, t_1)} \frac{\Delta_g(t_1, t_{\text{jet}})}{\Delta_g(t_{\text{jet}}, t_{\text{jet}})} \\
 \times & \frac{\Delta_g(t_1, t_{\text{jet}})}{\Delta_g(t_2, t_{\text{jet}})} P_{g \rightarrow gq}(z_2) \cdot \Delta_g^2(t_2, t_{\text{jet}}) \cdot d\hat{\sigma}_{q\bar{q}}(\hat{s})
 \end{aligned}$$

yields (for upper line only, 2 FS particles):

$$\mathcal{W}_{\text{corr.}} = \Delta_q(t, t_{\text{jet}}) \cdot \Delta_g(t_1, t_{\text{jet}}) \cdot \Delta_g(t_2, t_{\text{jet}})$$

Example : Drell-Yan



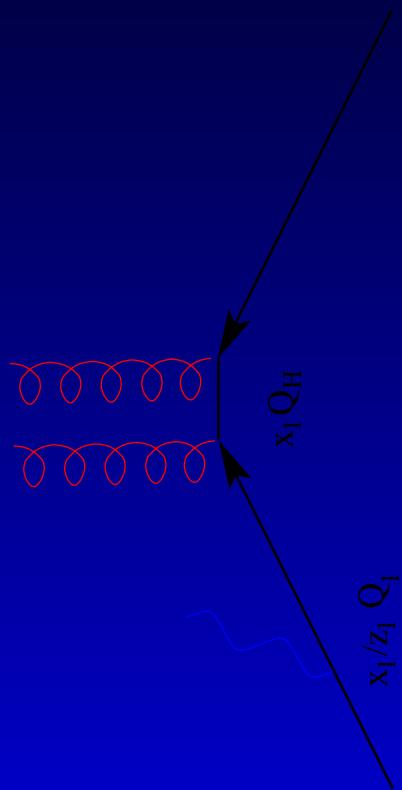
$$w_{PS} = \frac{f_q(x_1/z_1, t_1)}{z_1 f_q(x_1, t)} \frac{\Delta_q(t, t_{jet})}{\Delta_q(t_1, t_{jet})} \cdot P_{q \rightarrow qg}(z_1)$$
$$\times \frac{f_q(x_1/z_1, t_{jet})}{f_q(x_1/z_1, t_1)} \frac{\Delta_q(t_1, t_{jet})}{\Delta_q(t_{jet}, t_{jet})} \cdot d\hat{\sigma}_{q\bar{q}}(\hat{s})$$

yields (for upper line only, 0 FS particles):

$$\mathcal{W}_{\text{corr.}} = \Delta_q(t, t_{jet})$$

with different $t \sim M_{ll}^2 + p_{\perp,1}^2$.

Example : Drell-Yan



$$w_{PS} = \frac{f_q(x_1, t_{\text{jet}})}{f_q(x_1, t)} \frac{\Delta_q(t, t_{\text{jet}})}{\Delta_q(t_{\text{jet}}, t_{\text{jet}})} \cdot d\hat{\sigma}_{q\bar{q}}(\hat{s})$$

yields (for upper line only, 0 FS particles):

$$\mathcal{W}_{\text{corr.}} = \Delta_q(t, t_{\text{jet}})$$

with different $t \sim \max\{p_{\perp}^2(g)\}$.

Conclusions

1. Procedure proved for e^+e^- :
 - theoretically correct
 - practically correct
 - improved description of data
2. Being implemented for pp .
3. May be proved for ep .

AMEGIC++

- Calculation of arbitrary processes in the SM and the MSSM at the tree level using Helicity amplitudes

MSSM setup:

- Feynman rules (unitary gauge) according to J.Rosiek, Phys.Rev.D41:3464,1990
- Interface to ISASUSY to generate the MSSM spectra
- Majorana fermion Feynman rules as defined in Denner *et al.*, Nucl.Phys.B387,1992

AMEGIC++

- Calculation of arbitrary processes in the SM and the MSSM at the tree level using Helicity amplitudes
- Full massive matrix elements available
- Unstable particles treated via FWS or CMS
- Explicit external polarisations enabled
- Multi-channel MC integration with adaptive optimization
- Program can be run parallel using the MPI standard
- Completely automatic approach (a generator-generator)

Checks: 6f production

Comparison with the results of LUSIFER and WHIZARD
the later using the MEs of MADGRAPH

(S. Dittmaier and M. Roth, Nucl. Phys. B642 (2002))

- $\sqrt{s} = 500$ GeV, full SM, phase space cuts, massless fermions, 10^7 MC events

Top production channels (incl. QCD): (preliminary)

$e^+ e^- \rightarrow$	AMEGIC++ [fb]	LUSIFER [fb]	WHIZARD [fb]
$\mu^- \bar{\nu}_\mu \nu_\mu \mu^+ b \bar{b}$	5.8088(54)	5.8091(49)	5.8102(26)
$\mu^- \bar{\nu}_\mu \nu_\tau \tau^+ b \bar{b}$	5.8002(32)	5.7998(36)	5.7962(26)
$e^- \bar{\nu}_e \nu_\mu \mu^+ b \bar{b}$	5.8268(39)	5.8188(45)	5.8266(27)
$e^- \bar{\nu}_e \nu_e e^+ b \bar{b}$	5.8794(83)	5.8530(68)	5.8751(30)
$\mu^- \bar{\nu}_\mu u \bar{d} b \bar{b}$	17.209(9)	17.171(24)	–
$e^- \bar{\nu}_e u \bar{d} b \bar{b}$	17.329(11)	17.276(45)	–

AMEGIC++ calculation setup

Parameters:

G_μ	=	$1.16639 \times 10^{-5} \text{ GeV}^{-2}$	α_s	=	0.0925(0.0891) at 360(500) GeV
M_W	=	80.419 GeV	Γ_W	=	2.12 GeV
M_Z	=	91.1882 GeV	Γ_Z	=	2.4952 GeV
M_H	=	130.0 GeV	Γ_H	=	0.004291 GeV
M_μ	=	105.6583 MeV	M_τ	=	1.777 GeV
M_u	=	5 MeV	M_d	=	10 MeV
M_c	=	1.3 GeV	M_s	=	200 MeV
M_t	=	174.3 GeV	Γ_t	=	1.6 GeV
M_b	=	4.8 GeV			

Phase space cuts:

$$\begin{array}{ccccccc} \theta(l(q), \text{beam}) & > & 5^\circ & \theta(l, l') & > & 5^\circ & \theta(l, q) & > & 5^\circ \\ E_l & > & 10 \text{ GeV} & E_q & > & 10 \text{ GeV} & m(q, q') & > & 10 \text{ GeV} \end{array}$$

AMEGIC++ calculation setup

- Width treatment for unstable particles:

$$M_V^2 = m_V^2 - i m_V \Gamma_V, \quad V = W, Z$$

$$M_H^2 = m_H^2 - i m_H \Gamma_H, \quad M_t = m_t - i \Gamma_t / 2$$

$$D_F^{\mu\nu}(q) = \frac{-g^{\mu\nu} + q^\mu q^\nu / M_V^2}{q^2 - M_V^2}, \quad D_F(q) = \frac{1}{q^2 - M_H^2}$$

$$S_F(q) = \frac{q + M_t}{q^2 - M_t^2}$$

- The electroweak mixing angle is kept real (FWS):

$$\Rightarrow \sin^2 \theta_W = 1 - m_W^2 / m_Z^2$$

- Fermion masses are fully taken into account
- Higgs couplings to e, u, d have been neglected
- Results obtained after 10^6 Monte Carlo points

Tops (full-hadronic)

final state	\sqrt{s}	QCD	AMEGIC++ [fb]
$b\bar{b}u\bar{d}d\bar{u}$	360	yes	32.90(15)
	500	yes	49.74(21)
	360	no	32.22(34)
	500	no	49.42(44)
$b\bar{b}u\bar{u}g g$	360	–	11.23(10)
	500	–	9.11(13)
$b\bar{b}g g g g$	360	–	18.82(13)
	500	–	24.09(18)

SHERPA

1. A new event generator in C++
2. first modules ready:
 - State-of-the-art ME-generator
To be used by HERWIG++
 - Parton shower module
 - General ME+PS interface
3. Under construction:
 - General framework
(like Pythia7, but “lean”)
 - Hadronization
4. So far ≈ 80000 lines of code.

